RESEARCH ARTICLE

The size, multipartite Ramsey numbers for C₃ versus all graphs up to 4 vertices

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Abstract: In this paper we restrict our attention to finite graphs containing no loops or multiple edges. The multipartite graph $K_{i\times s}$ ($j \ge 3$) consisting of j partite sets of uniform size s is defined as $V(K_{j\times s}) = \{v_{mn} | m \in \{1, 2, ..., j\} \text{ and } n \in \{1, 2, ..., s\} \}$ and $E(K_{i\times s}) = \{v_{mn}, v_{kl} \mid m, k \in \{1, 2, ..., j\} \text{ and } n, l \in \{1, 2, ..., s\} \text{ where}$ $k \neq m$. The set of vertices of the m^{th} partite set is denoted by $\{v_{m} \mid n \in \{1, 2, \dots, s\}\}$. If for every two-colouring (red and blue) of the edges of a graph K, there exists a copy of H in the first colour (red) or a copy of G in the second colour (blue), we write $K \rightarrow (H,G)$. Given two simple graphs H and G, the Ramsey number r(H,G) is defined as the smallest positive integer s such that $K \rightarrow (H,G)$ and along the same line of reasoning, the multipartite Ramsey number $m_j(H,G)$ is defined as the smallest positive integer s such that $K_{j\times s} \rightarrow (H,G)$. Thus, multipartite Ramsey number $m_i(C_3, G)$ is defined as the smallest positive integer s such that any red-blue colouring of K_{ixx} contains a red C_{1} or a blue G. Since only a few multipartite Ramsey numbers for pairs of graphs have been found so far, in this paper we find all such multipartite Ramsey numbers $m_i(C_{\nu}G)$ when G is any graph up to 4 vertices.

Keywords: Combinatorics, graph theory, mathematics, multipartite Ramsey numbers, Ramsey theory.