



**ON STAR CRITICAL RAMSEY NUMBERS RELATED TO STARS
VERSUS $K_{1,m} + e$ FOR SMALL m**

Abstract. After the publication of the original paper by F. P. Ramsey, the new area of Graph Theory, namely Ramsey Theory, emerged with the contribution of the paper by Paul Erdős and George Szekeres, published around 1935. According to this paper, for any two graphs H, G and any red/blue colouring of K_n represented by $K_n = H_R \oplus H_B$, $K_n \rightarrow (H, G)$ if there exists a red copy H in H_R or a blue copy G in H_B . Using this notation, the Ramsey number $r(m, n)$ is defined as the smallest positive integer N such that $K_N \rightarrow (K_n, K_m)$. Erdős et al., proved that, for any $m \geq 2$ and $n \geq 2$, $r(m, n)$ exists and satisfies $r(m, n) \leq r(m - 1, n) + r(m, n - 1)$ and $r(m, n) \leq \binom{m + n - 2}{m - 1}$.

The exact determination of Ramsey numbers for small graphs has been attempted by many mathematicians in the subsequent years. In 2010, Hook and Isaak, introduced Star-critical Ramsey number $r^*(H, G)$ which captured the attention of many researchers in the recent years. The Star critical Ramsey number $r^*(K_{1,n-1}, G)$ for any simple graph G is defined as the largest integer k such that $K_{r(K_{1,n-1}, G)-1} \sqcup K_{1,k} \rightarrow (K_{1,n-1}, G)$. In this paper, starting with $G = K_{1,2} + e = K_3$, we exhaustively find Ramsey numbers $r(K_{1,n-1}, K_{1,m} + e)$ for $(1 \leq n \leq 5, 2 \leq m \leq 7)$ and subsequently based on this, we find Star critical Ramsey numbers $r^*(K_{1,n-1}, K_{1,m} + e)$ $(1 \leq n \leq 5, 2 \leq m \leq 7)$.

Keywords. Graph theory, Ramsey theory, Star critical Ramsey numbers.

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