Light travel time effect of the binary orbit of SZ Lyn

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Abstract

SZ Lyncis, HD 67390 (RA=08^h 09^m 35.8^s, DEC=+44° 28' 17.6") is a high amplitude Delta Scuti type binary star of magnitude m_v= 9.1, which has pulsation period of 0.12053491 days and long orbital period of 1185 days. We determine fifty two new times of light maxima and collected all times of light maxima to calculate the orbital elements of SZ Lyn. The photometric observations were carried out in six nights at Mount Abu Infrared observatory, India. The difference of the observed times of light maxima and calculated times of light maxima is denoted by the O–C diagram of SZ Lyn. A total of 378 light maxima, including our observations of 20 light maxima of SZ Lyn, 32 of Wide-angle search for planets (WASP), 162 of American Association of Variable Star Observers (AAVSO) and 164 observations that have been published, were used for the O–C analysis. The non linear O–C diagram was approximated by secular change in pulsation period and the light-travel-time effect of the binary orbit. The fitting non-linear function of secular change and light-travel-time effect in the least square method determines the orbital parameters projected semi-major axis of the binary orbit $a\sin(i)$, eccentricity (e), longitude of the periastron passage (ω), orbital period (P_{orbit}) and secular change of the pulsation period (β). The Levenberg-Marquardt algorithm and trust-region-reflective algorithm were used in the least square sense to converge theoretical function to observations with the minimization parameter χ^2 of the best fit. The determined $a\sin(i)$is $1.4\pm0.1\times10^8$ km and the eccentricity e is 0.18±0.07. The convergence of solution also approximated the orbital period to be 1187±15 days.

Key Words: *SZ Lyn, orbital parameters, photometric, pulsation period*

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1. INTRODUCTION

SZ Lyncis, HD 67390 (RA=08h 09m 35.8s, DEC=+44_ 280 17.600) is a high amplitude Delta Scuti type binary star, visual magnitude m_v= 9.1, spectral type of A7-F2, and has a pulsation period of 0.12053491 days and an orbital period of 1185 days (Soliman*et al*. 1986). The pulsating star is the brighter component while the faint component could not be observed in spectroscopy and is characterized as single line spectroscopic binary (Gazeas*et al.* 2004). The Delta Scuti stars are pulsators situated in the classical Cepheid instability strip on the main sequence or moving from the main sequence to the giant branch (Breger 2000). The main characteristic of Delta Scuti stars is the very short period pulsation time, ranging from 0.02 to 0.25 days. The orbital parameters of a binary system can be determined by spectroscopy if it were double line or by photometry if it were a transit system. SZ Lyn is neither double nor transit system. The pulsation property of SZ Lyn is used by observers, Van Genderen (1967), Moffett *et al.* (1975), Paparo*et al.* (1988), Li Lin-Jia*et al.* (2013), to determine orbital parameters with the use of O–C variations. The linear ephemeris of the time at maximum brightness for the pulsating stars is given as:

$$T_{max} = T_o + P \times E \quad (1)$$

where$E$ denotes cycle number, P is the pulsation period, T_o is the initial epoch of maximum and T_{max} is the time at maximum brightness of the observation. According to the equation 1, the ephemeris of the intrinsic fundamental pulsation period of SZ Lyn was first determined by Binnendijk (1968) as:

$$T_{max} = 2439121.7003 + 0.12053188 \times E \quad (2)$$

and it was redefined by Gazeas*et al.* (2004) as:

$$T_{max} = 2452776.289 + 0.1205349 \times E \quad (3)$$

The star has been discussed a number of times in the literature for the periodic variations and the main pulsation period is found to change $(2.25\pm0.42)\times10^{-12}$ day per cycle (Paparo*et al*. 1988). Van Genderen (1967) reported that the linear ephemeris does not accurately predict the time of pulsation maximum. Barnes and Moffett (1975) suggested that this was due to the very long period orbital motion of SZ Lyn and hence the linear ephemeris deviated due to the light-travel time across the orbit.

The times of light maxima predicted by the linear equation 1 are known as calculated light maxima and denoted by C. The observed times of light maxima were taken by the light curve of SZ Lyn, denoted as ‘O’. The assumption of constant pulsation period and linear ephemeris results in the O–C, the difference of observed and calculated times, being zero. The O–C diagram of SZ Lyn was previously studied by several authors, Moffett *et al*. (1988), Paparo*et al*. (1988) and showed a non-linear relation. The non-linearity of the O–C diagram can be explained by periodic variations of the main pulsation period and light-travel time across the binary orbit (Irwin 1952). The non-linear ephemeris is given by:

$$T_{max} = T_o + P \times E + \kappa + \tau \quad (4)$$

$$\kappa = \frac{\beta}{2} E^2 \quad (5)$$

$$\tau = \frac{a \sin(i)}{c}\left[\sqrt{1-e^2}\sin E^* \cos\omega + \cos E^* \sin\omega - e\sin\omega\right] \tag{6}$$

Equations (5) and (6) are the periodic variation and light travel time effect of the orbit respectively. $\beta$is the secular change in the pulsation period, $a\sin(i)$ is the projected semi-major axis to the line of sight, e is the eccentricity, E^* is the eccentric anomaly, ω is the longitude of the periastron passage and c is the speed of light. The combination of the intrinsic pulsation and binary orbit produces a very complex O–C diagram for SZ Lyn. A comprehensive O–C analysis provides details of the orbital parameters and the pulsation properties. The times of light maxima from Mount Abu observations, WASP (Wide-angle search for planets), AAVSO, and all the observations that have been published were used for the classical O–C analysis method. This paper reports the photometric observations carried out at Mount Abu observatory in Section 2, detailed analysis of O–C diagram in Section 3 and discussion and conclusions in Sections 4 and 5, respectively.

2. EXPERIMENTAL

The observations were carried out at Mount Abu Infrared observatory. The observatory is located 1680 meters above the sea level in the western state of Rajasthan, India. Observations were made using a 50 cm, f/6.8 equatorial mount telescope equipped with an Andor 1024x1024 EMCCD thermoelectrically cooled to -80 °C. A set of 4569 frames in the V band obtained over six nights were subjected to basic reduction steps of bias and flat field correction, and the instrumental magnitudes extracted by defining an aperture of four times the FWHM of the star. Though the field of view of the imaging system, 13×13 arc seconds, is relatively large, there were no stars of similar magnitude to SZ Lyn to perform the differential photometry. Therefore to normalize the six day observations, all magnitudes were shifted to the highest magnitude of six days. A part of normalized light curve is shown in Figure 1. The entire light curve provides 20 light maxima. In addition, the WASP observation of SZ Lyn in the V band of a total of 2894 frames covering 32 light maxima, 162 maxima from AAVSO and 164 from all the previous observations brought together for a total of 379 times of light maxima for O–C analysis.

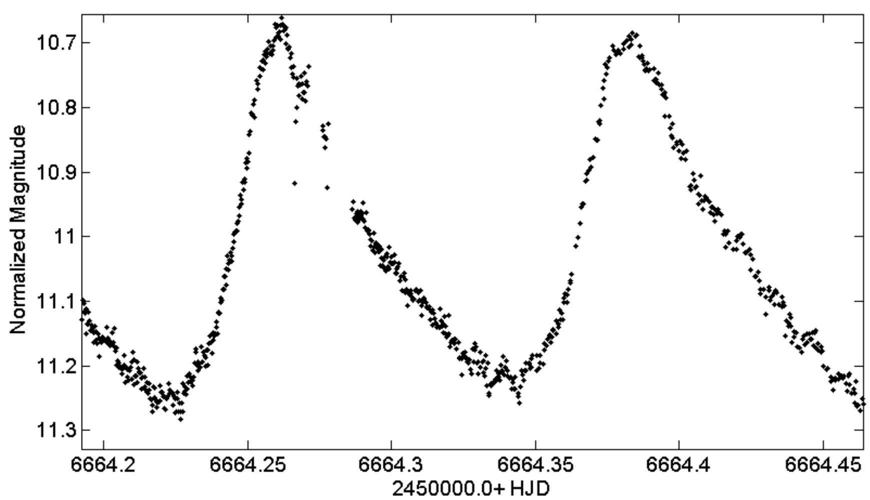

Figure1: Light curve of SZ Lyn taken on 6th January 2014 at Mount Abu observatory.

3. RESULTS & DISCUSSION

3.1 Observed times of light maxima (O)

The discrete magnitudes shown in Figure 1 were approximated by a continuous Fourier fitting and hence determine the first derivative of the function. The time equivalent to the zero point of the first derivative which is changing from negative to positive (Fig. 2) of the Fourier function was taken as the observed times of light maxima, ‘O’.

3.2 Calculated times of light maxima (C)

The ephemeris given by Paparo et al. (1998) in equation 7 was used to calculate the calculated times of light maxima (C). The number of cycles, E, were taken from equation 7 for the observed times of light maxima (T_m) and assuming pulsation period 0.12053492 days. These calculated values were converted to integers to represent the cycle number. Thereafter calculated times of lightmaxima, T_{max}which is denoted by ‘C’ were taken from equation 7 by feeding the corresponding number of cycles which are calculated using the observed T_{max}.

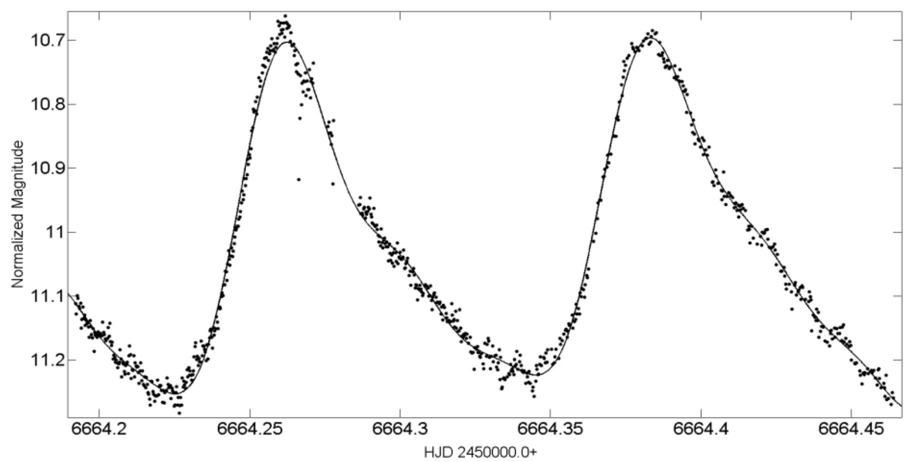

Figure 2 (a)

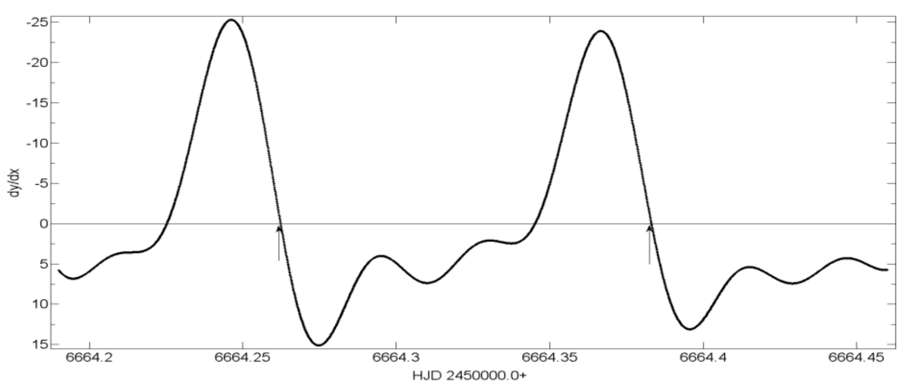

Figure 2 (b)

Figure 2: Fourier approximation of the light curve Figure 2(a). Note that the Y axis is reversed to get the correct magnitude scale. The first derivative of the Fourier function and the times of light maxima taken from zero points were marked with arrows Figure 2(b).

$$T_{max}\,(HJD) = \; 2438124.39955 + 0.12053492 \times E \qquad (7)$$

Finally the difference of observed and calculated times of light maxima were calculated and denoted as O–C. The O–C value against the cycle number E is shown in Figure 3.

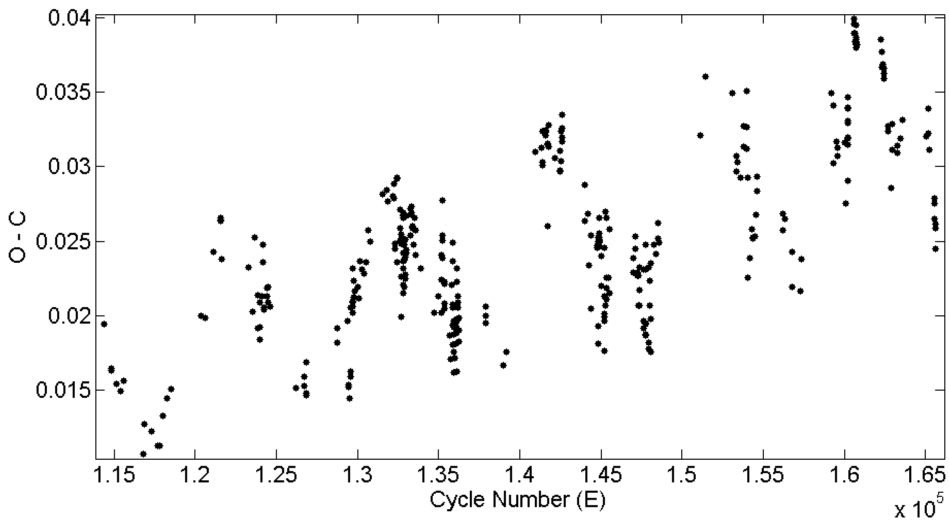

Figure 3: The difference of times of observed and calculated light maxima.

The O–C in Figure 3 is a non-linear distribution with an overall increase in O–C and some local variation. The non-linear behavior implied that there is a countable effect of light travel time in the binary orbit (Irwin 1952) as well as change in pulsation period. The overall increase is due to the change in pulsation period which is given by the quadratic equation in equation 5. The local variation is a sinusoidal function which is given by the light travel time effect of the binary orbit in equation 6. The introduction of light travel time effect to the O–C variation provides the orbital parameters to be included in the calculation. Therefore the equation 4 provides the unknown parameters for non-linear least square fitting to the O–C variation.

3.3 Non-linear curve fitting

The O–C diagram clearly shows that the O–C variation is not constant and it is non-linear. Therefore ephemeris in equation (1) is inadequate to explain the variation of O–C. The terms in equation 5 and 6 are additionally needed to explain the variation of O–C. The exponential term (κ) in equation 4 is resulted the overall increase of the O–C variation and the light travel time effect (τ) in equation 6 along the binary orbit is caused the sinusoidal variation. In order to fit the difference of observed and calculated times of light maxima, the equation of O–C should be included the correction values of T_o and P denoted by δT_o and δP respectively. Therefore O–C equation should be;

$$O - C = \; \delta T_o + \delta P \times E + \kappa + \tau \qquad (8)$$

The light time travel effect of equation 6 is a function of eccentric anomaly (E*). The eccentric anomaly should be converted to the common independent variable of cycle number (E) in equation 8. Therefore the equation 4 was transformed to the independent variable (E) using equations 9 and 10.

$$M = E^{*} - e \sin E^{*} \qquad (9)$$

$$M = \frac{2\pi}{P_{orbital}}(T_{max} - T) \quad (10)$$

where M is mean anomaly, $P_{orbital}$ is the orbital period of SZ Lyn, and T the time of passage through the periastron where E* is zero.

The parameters determined by Li Lin-Jia*et al*. (2013) given in Table 1 were used for initial calculations and estimations of δT_o and δP. The T_o and P values inTable 1 with the ephemeris given in equation 7 were used to calculate the δT_o and δP.

Table 1: Initial parameters of SZ Lyn used for optimization.

Parameter	Value
T_o (HJD)	2438124.39849
P (days)	0.120534908
β (days/cycle)	2.73×10^{-12}
a sin (i) (au)	1.002
e	0.17
ω (degrees)	117.6
P_{orbit} (days)	1182
T (days)	2445786.4

The optimization tool, *lsqcurvefit*, in MATLAB with two algorithms, 'trust-region-reflective' and 'Levenberg-Marquardt', was used iteratively to converge the solution. Equation 8 was fitted to the data points in the least square sense as shown in Figure 5 using *lsqcurvefit*solver. Two different optimization techniques given in table 2 determine the orbital parameters.

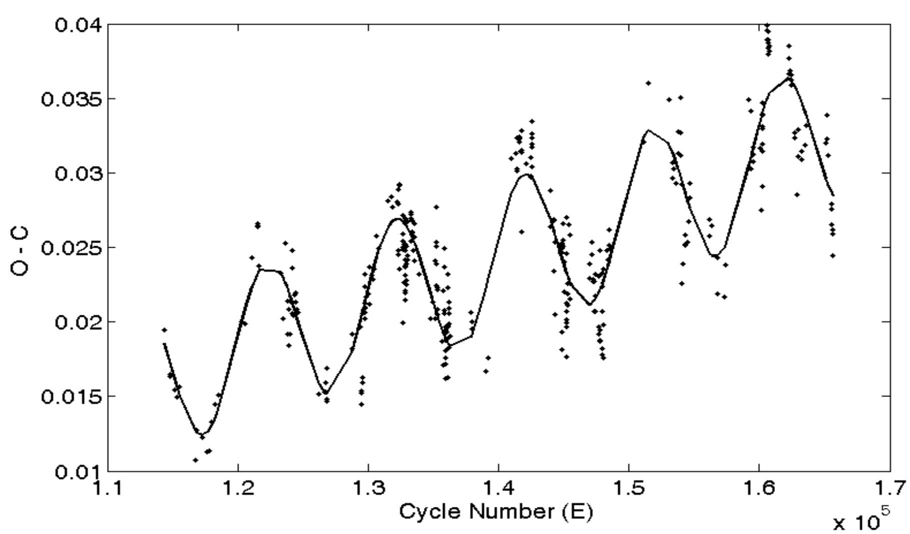

Figure 5(a)

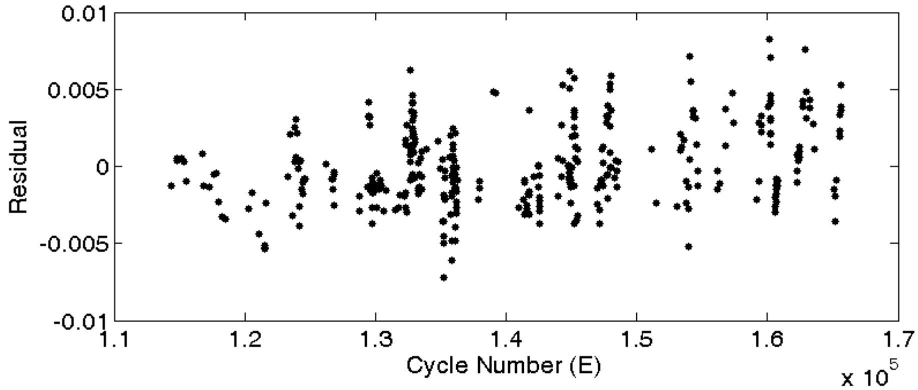

Figure 5(b)

Figure 5: The least square fitting of equation 8 to the observed O–C data Fig 5(a) and the residual of the function Fig 5(b).

We fixed the values of δT_o, δP, P and T as 0.001, 2×10^{-9}, 0.12053491 and 2454786.4 respectively and consider $a\sin(i)$, eccentricity (e), longitude of the periastron passage (ω), orbital period ($P_{orbital}$) and secular change (β) as coefficients in the non-linear equation. In thelsqcurvefit approach, the initial values for those coefficients were assigned and the solution was converged to the minimum value of χ^2,0.0022, in both algorithms mentioned in Table 2. Figure 5 shows the approximation of the equation 8 to the observations. The continuous line is the equation 8 with the secular change in the pulsation period and the light travel time effect of the orbital motion of the SZ Lyn.

Table 2: Optimization parameters of *lsqcurvefit*

Algorithm	Trust-region-reflective	Levenberg-Marquardt
MaxIter	5000	5000
TolFun	4×10^{-18}	4×10^{-18}
TolX	1×10^{-17}	1×10^{-17}
MaxFunEvals	5000	5000

Table 3: The obtained orbital and pulsation parameters of SZ Lyn

Parameter	Trust-Region	Levenberg- Marquardt
Status of Convergence	Local minima possible	Local minima found
χ^2	0.0022	0.0022
a sin(i) (km)	$(1.4\pm0.1)\times10^8$	$(1.4\pm0.1)\times10^8$
e	0.17±0.05	0.18±0.07
ω (°)	106±11°	106±5°
β (days/cycle)	$(2.4\pm0.4)\times10^{-12}$	$(2.2\pm0.2)\times10^{-12}$
P_{orbit}(days)	1186±15	1187±15

The errors of the coefficient were estimated by the*nlparci*function in MATLAB. The parameters, residual, lambda, and Jacobian (J) produced by *lsqcurvefit*function were fed into the *nlparci*function to get the 95% confidence intervals of the coefficients, which are given as the *ci* matrix.

4. DISCUSSION

SZ Lyn, the short period binary variable star, has been observed for many years. This binary system is very complicated as the major star, SZ Lyn, is pulsating in radial and non-radial modes. The light curve has the binary variation as well as the intrinsic pulsation variations. However this intrinsic pulsation time scale is very much shorter than the extrinsic magnitude variations due to the binary. The orbital parameters were calculated several times with the addition of times of light maxima contributed by different observers. Li Lin-Jia*et al*. (2013) have investigated the SZ Lyn binary system with the 262 times of light maxima. This study includes the AAVSO data for the O – C analysis providing a total number of 378 data points, the highest number of data points for an orbital analysis. The higher number of data points are significant for this analysis because it provides many orbital and pulsation cycles and hence the solutions are accurately converged. However the convergence is also depended on the quality of the observations. The AAVSO data is a collection of different observes with different systems. Therefore some inconsistency is shown in AAVSO data as the residual increases in Figure 5.

We used two algorithms to get the solutions. In Trust-region-reflective algorithm the, local minima are not found while in Levenberg-Marquardt (LM), the solution was converged. Therefore coefficients determined by LM method were considered as the final results. The handling of underdetermined systems using trust-region is unreliable. The optimization parameters in Table 2 were changed in Trust-region-reflective algorithm for the convergence, but the local minimacould not be found. Therefore it can be concluded that the LM method is more appropriate for a least squares solutions with several unknowns.

Although the O–C variation of SZ Lyn is well explained theoretically, it is very difficult to combine the intrinsic pulsation properties with the orbital properties as given in equations 8, 9, and 10. The solutions of the Kepler equation (equation 9), can only be obtained by successive approximations using Newton's method (Danby 2003). For simplicity we assumed the orbit is circular and therefore eccentricity e is zero. With this initial assumption of $e = 0$, the non-linear equation can be transferred to a common independent variable of cycle number (E). In this way we eliminate the eccentric anomaly (E*) from equation 8. But the binary system is non-circular and has some eccentricity, so the sine function fitted to the data points is not smooth enough as shown Figure 5.

The determined orbital parameters were agreed with the previous observations. Particularly the eccentricity *'e'* which is very important parameter for binary orbit is consistence for all the observations. The average value of all the observation is 0.19±0.02.

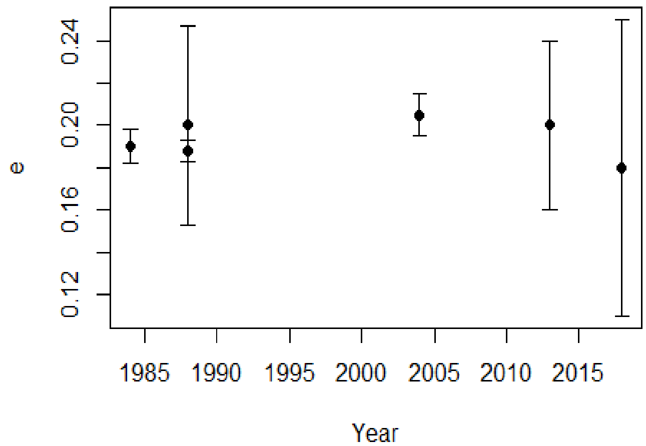

Figure 6: Thevariation of eccentricity of different observations.

The change in longitude of the periastron passage, also called apsidal motion in a binary system, has been observed for many years. This motion of the binary orbit can be caused by several effects such as general relativistic effect, tidal distortion, rotational flattening, a third component or combined effect (Li Lin-Jia*et al*. 2013). Li Lin-Jia (2013) proposed that ω is decreasing with time. In Figure 6, we found that the ω is inconsistently changing with the time.

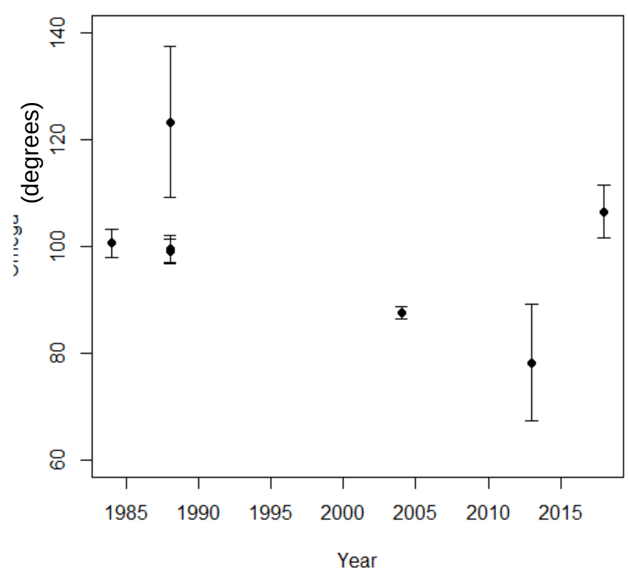

Figure 7: The variation of longitude of the periastron passage for seven different observers.

The dependency of the initial values and constants in the Table 1 to the determined orbital parameters was investigated. The value T, time of passage through the periastron where E*(T) = 0; determined by previous observers were different. These different values were used in the optimization process and it was found that there is no significant change in the χ^2 value of 0.0022.

5. CONCLUSIONS

The orbital parameters were determined with the new photometric observations of SZ Lyn with a total number of 378 light maxima. The O–C analysis determined the projected semi-major axis a sin (i) as $1.4\pm0.1\times10^8$ km and eccentricity, e is 0.18±0.07. The orbital period is determined as 1187±15 days. The orbital period is consistent within the above range, which does not deviate much from the previous determinations. The longitude of the periastron passage is highly uncertain as it is inconsistent in the previous determinations as well. Furthermore it can be concluded that the Levenberg-Marquardt least square algorithm is more appropriate for the convergence of a solution with several unknowns.

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