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Development of a mathematical model to evaluate the rate of aggregate risk of invasive alien plant species: Fuzzy risk assessment approach

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This study presents a novel approach to evaluate the rate of aggregate risk of Invasive Alien Plant Species. Using risk values and grade of importance of weights of risk factors which may reflect invasiveness of plant species are considered. We use Linguistic Ordered Weighted Averaging operator to evaluate the grade of important of weights. Since the risk values and important weights are identified from two different linguistic term sets, fuzzy set theory techniques were used to combine the two sets. The rates obtained from the model were compared with NRA risk levels and the model was validated with data from known and non-invasive species. The model is improved by weighting the risk values of risk factors. The improved model produced significant results and resulted a better tracking system for identifying potential invaders than the conventional risk assessment.

Keywords: Invasive alien plant species; invasive attributes; fuzzy set theory operators; linguistic ordered weighted averaging operator.

Mathematics Subject Classification 2010: 97R20, 92B99, 03E72

1. Introduction

Rate of introduction of Invasive Alien Plant Species (IAPS) is rapidly increasing worldwide and it is a major cause of global biodiversity loss and environmental change [12]. There is an urgent need for more rigorous and comprehensive risk analysis protocols for IAPS to prevent them and control invasions [3]. The Risk Assessment (RA) protocol is a common and simple tool which is being used to evaluate the invasion risk of species. This tool consists of questions related to risk factors of IAPS and generates an overall risk score which is the sum of scores that have been given to each question by domain experts. However, this procedure is not a clear cut process and the final output depends on the users' biasness. Further, RA requires opinions of group of expertise which is not always possible to organize at short notice.

Most of the problematic invasive plant species have a number of characteristics in common, including efficient seed dispersal, fast seed germination, growth spread through vegetative organs, etc. In RA, these characteristics are considered as risk factors which could reflect the invasiveness of species. Most of the risk factors are qualitative in nature i.e. not quantifiable. Hence, data for some risk factors have been gathered from knowledge of experts in plant sciences due to the unavailability or lack of proper mechanism to measure data [14]. For example it is very difficult to quantify the factor, vegetative reproductive strength (VRS) by using physical measurements. Hence the experts give their opinion using a linguistic scale consists of words like *Low*, *Medium*, *High*, etc., which should be large enough to provide their opinion.

We emphasize that integration of mathematical techniques is imperative to develop a comprehensive risk analysis. Among the existing techniques, fuzzy set theory can handle imprecise, uncertain situations than statistical tools because many cases accompanied with uncertainty cannot be solved using probability theories. Human perception on words may be different from person to person [7]. Therefore, aspects of fuzzy set theory are needed to capture the uncertainty of words used in RA.

In this paper, we propose a RA mathematical model to generate the rate of aggregate risk of IAPS. For this purpose most important invasive risk factors have been considered as parameters of the model to produce risk scores in the form of linguistic values. In many situations we can see that only the risk values are considered while neglecting the grade of importance of risk factors. In this work, not only its risk values, the grade of important weights of risk factors toward invasiveness have been incorporated.

We developed a method to aggregate the risk values and grade of important weights, which are coming from two different linguistic term sets. By incorporating this method, a model has been developed to generate the rate of aggregate risk of IAPS based on the algorithm proposed by Lee [11]. The model has been validated

Table 1. Invasive attributes.

Main risk factor	Subrisk factor
Dispersal (DIS)	
Growth (G)	Vegetative reproductive strength (VRS)
	Form dense thickets (FDT)
	Physical defensive structures (PDS)
Seed germination (SGR)	
Alleopathy property (ALP)	
Invasive races (IR)	
Man's influence on spreading (MIS)	Potential to be spread by human activities (HA)
	Role of natural and man made disturbances (NMD)

by testing a set of well-known invasive plant species and non-invasive species of Sri Lanka. The validation results of the model have been improved by introducing a weighting system for the risk values before aggregate with the grade of important weights. The validation results show that the improved model gives significant results of rate of aggregate risk of IAPS than that of conventional RA conducted for the same species.

2. Selection of Risk Factors

From scientific point of view the species invasiveness could be recognized by their invasive attributes. In this work, nine invasive attributes used for National Risk Assessment (NRA) for alien invaders in Sri Lanka have been selected as risk factors which are parameters of the model [13]. As presented in Table 1, the nine attributes have been categorized under main and subrisk factors. One may see that there are six main risk factors and only Growth and Man's influence on spreading have subrisk factors.

The dataset of known 33 IAPS and 10 non-invasive species was provided by the invasive specialist group attached to the Ministry of Environment and Renewable Resources, Sri Lanka. It contained linguistic-valued observations of 9 parameters and invasion risk scores of plant species of the NRA.

3. Assigning Grade of Important Weights

It is a known fact that the impact of a particular risk factor of invasiveness is different from factor to factor; even though these factors highly contribute to raise the invasive potential of species. Therefore, it is important to consider the grade of important weights along with the values of risk factors in imprecise uncertain situations.

In [2, 10], we can find some methods to evaluate important weights such as fuzzy analytical hierarchy process with chang's extent analysis, column geometric mean method, etc. or fuzzy set theoretic techniques. Here different techniques have been used to obtain the important weights in the form of linguistic terms. First, pairwise

Table 2. Linguistic scale for importance.

Label	Linguistic scale	Triangular fuzzy numbers
s_4	Absolutely more important	(0.75,1,1)
s_3	Very strongly more important	(0.5,0.75,1)
s_2	Strongly more important	(0.25,0.5,0.75)
s_1	Equally important	(0,0.25,0.5)
s_0	Weakly more important	(0,0,0.25)
	Just equal	(1,1,1)

comparison of subrisk factors with respect to their main factor and pairwise comparisons among six main factors toward invasion risk have been obtained. A questionnaire form was constructed to obtain the decision makers pairwise comparisons among the model parameters (see Appendix A) and three plant science experts provided their pairwise comparisons in order to obtain the important weights using the linguistic scale which is presented in Table 2 [11].

3.1. *Evaluating grade of importance of weights*

- After collecting the pairwise comparisons the final task was to aggregate the responds. Here our aim was to aggregate linguistic terms without using their fuzzy numbers. Hence corresponding linguistic term set was matched with their membership functions as depicted in Fig. 1.
- The operator Linguistic Ordered Weighted Averaging (LOWA) has been chosen among the aggregation operators of linguistic non-weighted information to aggregate the responds [9]. The LOWA operator is a symbolic operator with properties like increasingly monotonous, commutative and "or-and" operator.
- We reproduce the Definition 3.1 as given in [9].

Definition 3.1. Let $A = \{a_1, \ldots, a_m\}$ be the set of labels to be aggregated, then the LOWA operator, ϕ, is defined as

$$\begin{aligned}\phi(a_1, \ldots, a_m) &= W \cdot B^T = \xi^m\{w_k, b_k, k = 1, \ldots, m\} \\ &= w_1 \cdot b_1 \oplus (1 - w_1) \cdot \xi^{m-1}\{\beta_h, b_h, h = 2, \ldots, m\},\end{aligned}$$

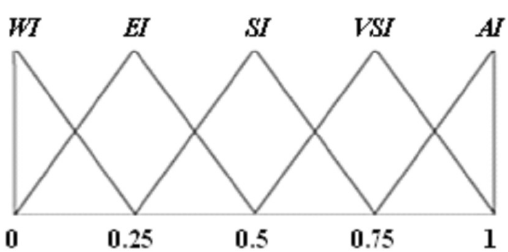

Fig. 1. Membership functions of linguistic scale for importance.

where $W = [w_1, \ldots, w_m]$ is a weighting vector, such that (i) $w_i \epsilon [0,1]$, (ii) $\Sigma_i w_i = 1$, $\beta_h = w_h / \Sigma_2^m w_k, h = 2, \ldots, m$, and $B = \{b_1, \ldots, b_m\}$ is a vector associated to A, such that $B = \sigma(A) = \{a_{\sigma(1)}, \ldots, a_{\sigma(1)}\}$ in which $a_{\sigma(j)} \leq a_{\sigma(i)} \forall i \leq j$, with σ being a permutation over the set of labels A. ξ^m is a convex combination operator of m labels and if $m = 2$, then it is defined as

$$\xi^2\{w_i, b_i, i = 1, 2\} = w_1 \cdot s_j \oplus (1 - w_1) \cdot s_i = s_k, s_i, s_j \epsilon S (j \geq i),$$

such that $k = \min\{T, i + \text{round}(w_1 \cdot (j - i))\}$, where "round" is the usual round operation, and $b_1 = s_j$, $b_2 = s_i$.

Similarly as mentioned in [9] the weights W represent the concept of fuzzy majority [6] in the aggregation of LOWA operator using fuzzy linguistic quantifier [15, 17]. Zadeh suggested that the semantic of a linguistic quantifier can be captured by using fuzzy subsets for its representation [16]. The method proposed by Yager has been used to calculate the weights by means of fuzzy linguistic quantifier, in the case of a non-decreasing proportional fuzzy linguistic quantifier Q [8]:

$$w_i = Q(i/n) - Q((i-1)/n), \quad i = 1, \ldots, n,$$

Being the membership function of Q, it is as follows:

$$Q(i/n) = \begin{cases} 0 & \text{if } r < a, \\ (r-a)/(b-a) & \text{if } a \leq r \leq b, \\ 1 & \text{if } r > b \end{cases} \tag{3.1}$$

with $a, b, r, \epsilon [0, 1]$ and $r = (i/n)$. Some examples of quantifiers which have been used in this work are shown in Fig. 2, where the parameters (a, b) are $(0.3, 0.8)$, $(0, 0.5)$ and $(0.5, 1)$, respectively.

For example, consider the weighting vector obtained for the aggregation using "As many as possible" quantifier guider with the pair $(0.5, 1)$:

$$w = \left[\frac{2}{3}, \frac{1}{3}, 0\right], \quad \text{where}$$

$$w_1 = Q(1/3) - Q(0/3) = 0,$$

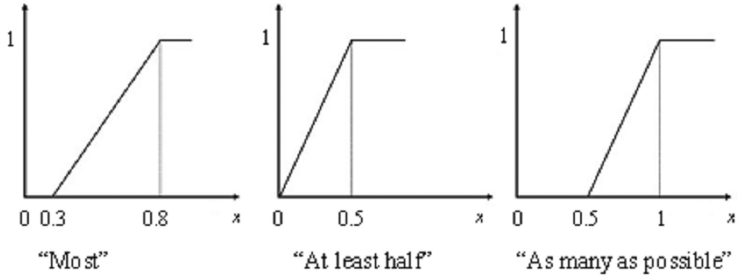

Fig. 2. Fuzzy linguistic quantifiers.

Table 3. Grade of important weights of risk factors.

Main risk factor	Grade of important weight	Subrisk factor	Grade of important weight
DIS	Very strongly more important		
G	Very strongly more important	*VRS*	Very strongly more important
		FDT	Very strongly more important
		PDS	Strongly more important
SGR	Strongly more important		
ALP	Strongly more important		
IR	Very Strongly more important		
MIS	Equally important	*HA*	Very strongly more important
		NMD	Weakly more important

$$w_2 = Q(2/3) - Q(1/3) = \frac{\frac{2}{3} - 0.5}{1 - 0.5} - 0 = \frac{1}{3},$$

$$w_3 = Q(3/3) - Q(2/3) = \frac{1 - 0.5}{1 - 0.5} - Q(2/3) = 1 - \frac{1}{3} = \frac{2}{3}.$$

The grade of important weights for main risk factors and their subrisk factors has been evaluated using LOWA operator with different linguistic quantifiers. Table 3 illustrates the important weights of main risk/subrisk factors evaluated with "As many as possible" linguistic quantifier guider.

One may note that LOWA operator has been applied twice in this process. First we aggregated the responds using LOWA for each main risk/subrisk factor in comparison to other main risk/subrisk factor. Again applied it as a linguistic choice function [9] to choose the best possible value for the important weight.

For instance, consider the subrisk factor *VRS* of main factor growth. The pairwise comparisons of VRS with respect to remaining subfactors of growth obtained from three experts are as in Table 4.

The labels in same column of Table 4 are aggregated by LOWA with "As many as possible" linguistic quantifier guider. Now consider the labels of the column FDT with respect to VRS: $\{(\text{Expert 1}, \mathit{VSI}), (\text{Expert 2}, \mathit{EI}), (\text{Expert 3}, \mathit{SI})\}$. The weighting vector is $w = \left[\frac{2}{3}, \frac{1}{3}, 0\right]$. By preparing the labels in descending order we have (*VSI*, *SI*, *EI*). First consider the pair *SI* and *EI*. Appling LOWA

$$k_2 = \min\{4, 1 + r(1 \times 1)\} = SI \quad (\text{here } j = 2 \text{ and } i = 1).$$

Table 4. Expert's pariwise comparisons on *VRS* with respect to *FDT* and *PDS*.

		VRS	*FDT*	*PDS*
Expert 1	*VRS*	*JE*	*VSI*	*VSI*
Expert 2	*VRS*	*JE*	*EI*	*EI*
Expert 3	*VRS*	*JE*	*SI*	*EI*

Table 5. Aggregated pariwise comparisons of *VRS* with respect to *FDT* and *PDS*.

	VRS	*FDT*	*PDS*
VRS	—	*VSI*	*SI*

For the last pair *VSI* and *SI*

$$k_3 = \min\left\{4, 3 + r\left(\frac{2}{3} \times 1\right)\right\} = VSI \quad \text{(here } j = 3 \text{ and } i = 2\text{)}.$$

Following the same procedure the final labels of the remaining columns may be obtained. Table 5 represents the final column labels.

Again the two labels (*VSI*, *SI*) aggregate using LOWA with "As many as possible" quantifier guider with weighting vector $w = [1, 0]$. Therefore, the important weight of *VRS* is *VSI*.

4. Structure of the Model

First we denote the symbols of the components of the model as follows:

$R(TAR_i)$ — Rate of total aggregate risk of ith plant species,
GW_{Mj} — Grade of important weight of jth main risk factor (numerical value),
$(RM_j)_i$ — Risk value of jth main risk factor of ith plant species (fuzzy number of particular linguistic term),
$(RS_m)_i$ — Risk value of mth subrisk factor of ith plant species (fuzzy number of particular linguistic term),
W_{Sm} — Grade of important weight of mth subrisk factor (fuzzy number of particular linguistic term),
AR_i — Aggregation of W_{Sm} and $(RS_m)_i$,
$(SAAR_j)_i$ — Risk value of jth main risk factor by aggregating corresponding risk of subrisk factors.

Here we expect that the final rates of risks emerge from the domain of the seven linguistic labels set as in Fig. 3.

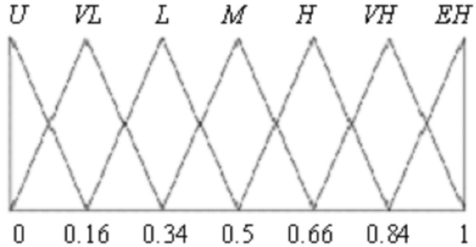

Fig. 3. Membership functions of linguistic term set L.

4.1. *Calculating AR_i*

For a particular ith plant species AR_i is defined as

$$AR_i = d(W_{Sm} \otimes (RS_m)_i), \tag{4.1}$$

where d denotes the centroid method which is one of defuzzification methods and $\otimes$ denotes the usual multiplication of normal triangular fuzzy numbers.

Now consider the linguistic term set L; L_0 — *Unlikely*, L_1 — *Very Low*, L_2 — *Low*, L_3 — *Medium*, L_4 — *High*, L_5 — *Very High*, L_6 — *Extremely High* as in Fig. 3. Considering Fig. 3. we take x as AR_i and obtain the intersection of AR_i and $\mu(x)$ as $L_m(1, 2, \ldots, k), k = 1, 2, \ldots, 7$. Here the functions μs are the membership functions of seven linguistic labels. For example if $AR_i = 0.185$ then $L_m(0, 0, \ldots, 0) = (0, 0.89, 0.11, \ldots, 0)$. Now define $(SAAR_j)_i$ as the risk of jth main risk factor by aggregating the risk values of subrisk factors with corresponding important weights.

$$(SAAR_j)_i = \Sigma_m L_m(1, 2, \ldots, k), \tag{4.2}$$

where $\sum_{k=1}^{7}(SAAR_j)_i = 1$.

One may note that if a risk factor itself is a main risk factor, in that case $(SAAR_j)_i = (RM_j)_i$.

Remaining part of this model is to evaluate the overall aggregate risk by aggregating GW_{Mj} with $(SAAR_j)_i$.

The important weights evaluated for each factor as in Table 3 have been defuzzified using center of gravity method [1]. Here the linguistic quantifier guider is "As many as possible" and results are represented in Table 6.

The overall risk TAR_i for ith species is calculated as follows.

Considering the risk value and GW_{Mj} for each main factor

$$\begin{aligned} TAR_i &= GW_{Mj} \otimes (SAAR_j)_i \\ &= (GW_{M1}, GW_{M2}, \ldots, GW_{Mm}) \otimes (SAAR_j)_i \\ &= (TAR_1, TAR_2, \ldots, TAR_7)_i. \end{aligned} \tag{4.3}$$

To evaluate the rate of aggregate risk, defuzzify TAR_i using centroid method as below:

$$R(TAR_i) = \frac{\sum_{k=1}^{7} GV(k) \otimes TAR_i}{\sum_{k=1}^{7} TAR_i}. \tag{4.4}$$

Table 6. Grades of important weights of main risk factors.

Main risk factor	GW_{Mj}
Dispersal	0.214286
Growth	0.214286
Seed germination requirement level	0.142857
Alleopathy property	0.142857
Invasive races (IR)	0.214286
Man's influence on spreading	0.071429

Here $GV(k)$s are the values of linguistic term set L defuzzified by the centroid method. Since the NRA scores are coming from a different manually conduct system, these were transformed into linguistic labels in set L. A numerical-linguistic transformation function has been used for this conversion [4, 5].

5. Results I

The proposed method has been applied to evaluate the rate of aggregate risk of 27 known invasive species in Sri Lanka. These rates have been compared with the risk levels given in NRA [16]. The results are tabulated in Table 7.

The model has been validated using few known invasive and non-invasive species and results are displayed in Table 8.

6. Discussion I

According to Table 7 it can be clearly seen that the rates of aggregate risks of species which take NRA risk level *High* or above appear in the range of 0.2655–0.5722.

For the species with *Medium* NRA risk level risk rates range from 0.197 to 0.4487. When NRA risk level is *Low* the species range of rate of risk has been 0.174–0.3267. The results are depicted in Fig. 4.

Table 7. Test results of Model I.

Invasive species	Risk level (NRA)	Rate of aggregate risk
Mikania micrantha	*High*	0.572154395
Chromolaena odorata	*High*	0.568820784
Mimosa pigra	*Very High*	0.555955808
Pennisetum polystachion	*High*	0.531504325
Leucaena leucocephala	*High*	0.52704379
Colubrina asiatica	*Medium*	0.448739556
Sphagneticola trilobata	*Medium*	0.422682288
Miconia calvescens	*High*	0.350024798
Mimosa invisa	*High*	0.337120182
Ageratina riparia	*Medium*	0.336364329
Clusia rosea	*Low*	0.326705113
Prosopis juliflora	*High*	0.325612544
Opuntia stricta	*High*	0.324165436
Parthenium hysterophorus	*Medium*	0.316936045
Lantana camara	*High*	0.313184682
Tithonia diversiflora	*Medium*	0.283112453
Myroxylon balsamum	*Low*	0.278853243
Alstonia macrophylla	*Low*	0.273461163
Imperata cylindrical	*High*	0.265542444
Ziziphus mauritiana	*Medium*	0.262612882
Clidemia hirta	*Medium*	0.255185359
Eichhornia crassipes	*Medium*	0.240426202
Pistia stratiotes	*Medium*	0.240426202
Alternanthera philoxeroides	*Medium*	0.23174
Ulex europaeus	*Medium*	0.216863667
Annona glabra	*Medium*	0.196959
Dillenia suffructicosa	*Low*	0.173985

Table 8. Validation results of Model I.

Category of species	Invasive species	NRA	Model I
Invasive	*Austroeupatorium inulifolium*	*High*	0.489959213
	Panicum maximum	*High*	0.470406151
	Cuscuta campestris	*High*	0.351996169
	Pueraria montana	*Medium*	0.23217039
	Acacia mearnsii	*High*	0.549982137
	Myrica faya	*Low*	0.229461789
Non-invasive	*Cassia fistula*	*Low*	0.191124
	Cissus rotundifolia	*Low*	0.195174
	Hedychium gardnerianum	*Low*	0.180057937
	Magnefera indica	*Low*	0.172286499

Figure 4 shows how the model output intersect with the NRA risk levels or the expected risk levels. It can be seen that the range of model output of low risk level plant species is a subset of low NRA risk level.

On the other hand the left end of the model output range for *Medium* risk is point 0.147 behind the expected left end of the *Medium* risk level. This has been resulted for some species such as *Ulex europaeus*, *Annona glabra*, *Alternanthera philoxeroides*, *Pueraria montana*, where the output is little behind from 0.34.

But the right end of the output range is within the expected level. One may see that the left end of the model output range is point 0.2345 behind the left end of the expected level but the right end is within the specific range.

The deviation here has been occured only for some species such as *Mimosa invisa*, *Prosopis juliflora*, *Opuntia stricta*, *Lantana camara* and *Imperata cylindrical*. However, as per the validation results all the non-invasive species rates are low and found within the expected risk level.

There may be some specific reasons for the deviations occurred in this evaluation. One major reason could be the evaluation of grade of importance. In this evaluation we have gathered the plant science experts' pairwise comparisons among the main risk/subrisk factors. In reality the optimism level may change from one decision maker to another decision maker on a particular task.

In order to minimize the deviations the model has been improved by weighting the risk values of risk factors as in Sec. 7.

7. Improved Model — Model II

As we discussed in Sec. 5 the deviations among the expected risk level and actual risk level arise due to specific reasons. The model aggregates only the risk values and grade of important weights of risk factors. On the other hand risk values are the linguistic labels which are almost approximations. Therefore, the real impact of risk factors may not generate from the model. Hence a new weighting mechanism was introduced to weight the risk values as follows.

Table 9. Weights (W') for main risk and subrisk factors.

Main risk factor	W'_M	Subrisk factor	W'_s
DIS	*Very High*		
G	—	*VRS*	*Very High*
		FDT	*High*
		PDS	*Very High*
SGR	No weight assigned		
ALP	*Very High*		
IR	*Very High*		
MIS	—	*HA*	*High*
		NMD	*High*

Let us denote W' as new weights given for selected main risk/subrisk factors. The W's take the form of linguistic labels as in Fig. 2. To find the suitable W's, randomly selected W's are taken from set L aggregated with each $(RS_m)_i$ and $(RA_j)_i$ under different linguistic quantifier guiders. These weighted risk values of $(RS_m)_i$ and $(RA_j)_i$ are tested with Model I. The final weights W' for main factors/subfactors are the values that may give slightest deviation among model output and NRA risk levels. One may note that except the weighted risk values of main risk/subrisk factors other steps of Models I and II are same. The final weights W' are shown in Table 9. For example the species *Cluis rosea*'s risk value for *VRS* is *Very Low* and according to Table 7 the weight for *VRS* is *Very High*. If we apply LOWA operator to aggregate these two values under mean quantifier guider the weighted risk value of *VRS* of *Cluis rosea* is *Medium*. The final value $R(TAR_i)$ has been matched with the linguistic term sets in Fig. 2 to make the comparison with NRA risk level process easier. For that task the numerical-linguistic transformation function has been used [16, 17].

After analyzing the results the model was decomposed into four cases as follows:

Case I: If a species $VRS \leq$ *Low* and $PDS =$ *Unlikely* and $FDT =$ *Unlikely* or $DIS \leq$ *Low* and $VRS \leq$ *Low* and $PDS \leq$ *Unlikely* and $AP =$ *Unlikely* and $IR =$ *Unlikely* then use Model I. Here weights do not assign for main risk and subrisk factors.

Case II: If a species $VRS \geq$ *Medium* and $PDS =$ *Unlikely* and $FDT =$ *High* and $AP =$ *High* and $IR =$ *Unlikely* then risk values for *VRS*, *FDT*, *NMD*, *DIS*, *IR* aggregate with W' as in Table 9 with mean quantifier guider.

Case III: If a species does not belong either above case then risk values for *VRS*, *FDT*, *NMD*, *DIS*, *IR*, *AP* aggregate with W' as in Table 9 with mean quantifier guider.

The test results are represented in Table 10. One may note that the rates of risk were tallied with set of linguistic terms L using the transformation function in [15] to make the analysis easier. Moreover, the model has been validated using the same set of data which used to validate Model I and the results are shown in Table 11.

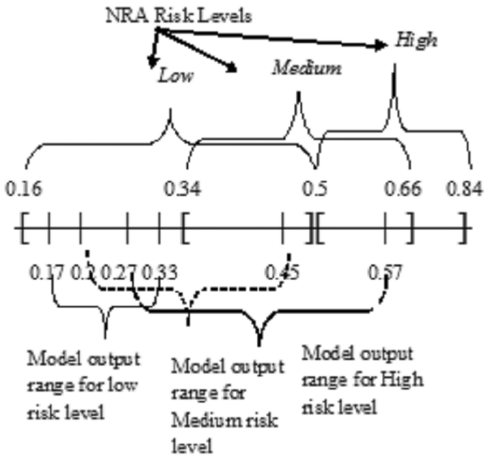

Fig. 4. Comparison of Model I output with NRA risk levels.

Table 10. Test results of improved model — Model II.

Invasive species	Risk level (NRA)	Rate of aggregate risk	Linguistic labels of rate of aggregate risk
Alternanthera philoxeroides	*Medium*	0.525779	*Medium*
Clidemia hirta	*Medium*	0.541375	*Medium*
Miconia calvescens	*High*	0.596138	*High*
Alstonia macrophylla	*Low*	0.273461	*Low*
Annona glabra	*Medium*	0.477566	*Medium*
Clusia rosea	*Low*	0.326705	*Low*
Dillenia suffructicosa	*Low*	0.445421	*Medium*
Ageratina riparia	*Medium*	0.610195	*High*
Mimosa invisa	*High*	0.604604	*High*
Myroxylon balsamum	*Low*	0.278853	*Low*
Tithonia diversiflora	*Medium*	0.588769	*High*
Mikania micrantha	*High*	0.700198	*High*
Prosopis juliflora	*High*	0.62734	*High*
Ulex europaeus	*Medium*	0.496612	*Medium*
Mimosa pigra	*Very High*	0.680319	*High*
Chromolaena odorata	*High*	0.700198	*High*
Parthenium hysterophorus	*Medium*	0.578769	*Medium*
Lantana camara	*High*	0.577937	*Medium*
Imperata cylindrical	*High*	0.587697	*High*
Opuntia stricta	*High*	0.415049	*High*
Colubrina asiatica	*Medium*	0.595311	*High*
Pennisetum polystachion	*High*	0.608643	*High*
Sphagneticola trilobata	*Medium*	0.663411	*High*
Ziziphus marutiana	*Medium*	0.592565	*Medium*
Eichhornia crassipes	*Medium*	0.543293	*Medium*
Pistia stratiotes	*Medium*	0.534714	*Medium*
Leucaena leucocephala	*High*	0.534714	*High*

Table 11. Validation results of Model II.

Category of species	Invasive species	Risk level (NRA)	Rate of aggregate risk	Linguistic labels of rate of aggregate risk
Invasive	*Austroeupatorium inulifolium*	*High*	0.627487	*High*
	Panicum maximum	*High*	0.636772	*High*
	Cuscuta campestris	*High*	0.63324	*High*
	Pueraria montana	*Medium*	0.570898	*Medium*
	Acacia mearnsii	*High*	0.702725	*High*
	Myrica faya	*Low*	0.191124	*Medium*
Non-invasive	*Cassia fistula*	*Low*	0.195174	*Very Low*
	Cissus rotundifolia	*Low*	0.180058	*Very Low*
	Hedychium gardnerianum	*Low*	0.542407	*Very Low*
	Magnefera indica	*Low*	0.172286	*Very Low*

8. Discussion II

Table 10 clearly shows how the rates of aggregate risks distribute on expected/NRA risk levels. The species rate of risk is in the range of 0.273461–0.445421 if it takes NRA risk level *Low*. If NRA risk level is *Medium* the species range of rate of risk is between 0.415049 and 0.610195. On the other hand the rate of aggregate risk of species which takes NRA risk level *High* is in the range of 0.577937–0.7. One may note that only one species shows *Very High* in NRA risk levels and it takes 0.680319 for rate of aggregate risk. Figure 5 shows how the model output ranges intersect with the NRA risk levels or the expected risk levels. It can be clearly seen that most of species rate of risk are within the expected risk level. The discrimination occurred in expected risk level and the ranges of rates of risk in Model I have been clearly solved in Model II. This can be proven more by the validation results. The rates of

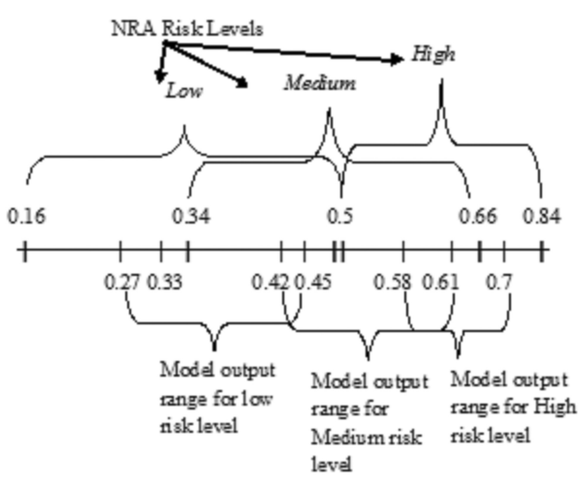

Fig. 5. Comparison of Model II output with NRA risk levels.

invasive species in the validation list are within the expected risk levels. The most highlighting part is the rates of non-invasive species because normally these species perform on invasive potential are considerably low. Here the validation results show that these species belong to *Very Low* risk level even though in NRA these are in Low category.

9. Conclusion

In this paper, two models with linguistic inputs have been constructed to assess the risk of IAPS. We have used fuzzy linguistic aggregation operators to aggregate the pairwise comparisons without using the fuzzy numbers. Here the grades of important weights of risk factors are aggregated with non-weighted risk values in Model I and with weighted risk values in Model II. The proposed models gave better prediction of risks of IAPS when invasion is dominated by invasive attributes. It is also worth mentioning that Model II has produced significantly improved results in comparison to Model I and provides a better prediction than the conventional risk assessment method (NRA). The models need to be modified by incorporating the risk factors other than invasive attributes, e.g. ecology, establishment, management aspects, etc., to evaluate the overall invasion risk. But the limited amount of available data on those factors set serious constraints for evaluation of overall risk of IAPS.

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Appendix A. Questionnaire for Collecting Pairwise Comparisons of Risk Factors

- This questionnaire was specially designed to perform all of possible pairwise comparisons among risk factors.
- Table A.1 shows the linguistic scale. You may follow the scale to indicate the important ratio of one factor to another i.e. comparison among main factors and subfactors within each main factor.

Let us illustrate an example of comparison among main risk factors and subfactors using linguistic scale (i.e. risk category $1 \rightarrow 2$). For instance, assume that the importance of main factor A compared to main factor B is strongly more important toward the invasiveness then assign 3 in corresponding cell in Fig. A.1. Assume that the importance of main factor B compared to main factor A is weakly more important then assign 2 in the corresponding cell in Fig. A.2.

Table A.1. Linguistic scale for importance.

Intensity of importance	Definition
5	Absolutely more important
4	Very strongly more important
3	Strongly more important
2	Equally important
1	Weakly more important
0	Just equal

Main factors		Risk category 2					
		A	B	C	D	E	F
Risk category 1	A	0	3				
	B		0				
	C			0			
	D				0		
	E					0	
	F						0

Fig. A.1. Comparison I — Main factors.

Main factors		Risk category 2					
		A	B	C	D	E	F
Risk category 1	A	0					
	B	2	0				
	C			0			
	D				0		
	E					0	
	F						0

Fig. A.2. Comparison II — Main factors.

			Risk Category 2				
Risk Category 1	Sub Factors		A			F	
			A1	A2	A3	F1	F2
	A	A1	0	5			
		A2		0			
		A3			0		
	F	F1				0	
		F2					0

Fig. A.3. Comparison I — Subfactors.

			Risk Category 2				
	Sub Factors		A			F	
			A1	A2	A3	F1	F2
Risk Category 1	A	A1	0				
		A2	3	0			
		A3			0		
	F	F1				0	
		F2					0

Fig. A.4. Comparison II — Subfactors.

Now let us consider that the importance of subfactor A_1 compared to subfactor A_2 is absolutely more important toward the main factor A then assign 5 in corresponding cell in Fig. A.3. Assume that the importance of subfactor A_2 compared to subfactor A_1 is strongly more important then assign 3 in the corresponding cell in Fig. A.4.

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